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6. AUTHOR(S) B.L. Rozovskii and F. LeGland				
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13. ABSTRACT (Maximum 200 words) The final report contains the outline of the research that was done during the period 1995-98. The main objective was to develop effective numerical algorithms of optimal nonlinear filtering and prediction and (more generally), state and parameter estimation in partially observed stochastic dynamical systems. During the course of the project a number of fundamental results were obtained, such as: development of a Wiener type optimal nonlinear filter (complete solution of "the last Wiener problem"); development of the spectral based approach to nonlinear filtering, which have led to the spectral separating scheme (separation of parameters and observations in optimal nonlinear filter) and other effective numerical approximations for the optimal nonlinear filter that include projection filter and assumed density filters. The results have been applied to specific "difficult" problems in target tracking, particularly, to the angle only tracking in EO and IR search and track systems and track-before-detect of resolved or sub-resolved low SNR targets. Extensive simulation showed that the proposed approach allows us to obtain much better performance as compared to the conventional expended Kalman filter in a number of important practical situations.				
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Nonlinear Filtering
Stochastic Analysis and Numerical Methods

Final Progress Report

B.L. Rozovskii and F. LeGland

Period 04/01/95 – 03/31/98

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Nonlinear Filtering: Stochastic Analysis and Numerical Methods

1 Statement of the Problem

The project was concerned with the analysis of partially observed stochastic differential systems and, in particular, with numerical methods in nonlinear filtering. The major objective was to develop numerical methods that could be used in a variety of state estimation problems and in associated statistical problems. The research was conducted by a team of investigators with expertise in nonlinear filtering, SPDE's, numerical analysis, and statistics.

The thrust of the research during the reporting period was in numerical aspects of nonlinear filtering, in particular in development of numerical approximation schemes suitable for applications in real time target tracking.

Filtering (estimation of a "state process" from noisy observations) is a classical problem in the statistics of stochastic processes. It is of central importance in navigation, image and signal processing, control theory, automatic tracking systems, and other areas of engineering and science. Filtering is one of the exemplary areas where the application of modern stochastic analysis and stochastic numerics lead to substantial advances in engineering.

Target tracking and identification is one of the main application of nonlinear filtering and will be emphasized in examples below.

The desired solution of the filtering problem is an algorithm that provides the best mean square estimate of the given functional of the state process. In many applications (most notably, battlefield target tracking) real time implementation of such algorithm is an important requirement. In the Gaussian case, the optimal (Kalman) filter [39] meets this requirement. The real time implementation of Kalman filter is readily available even for higher dimensional state processes due to the fact that in the Gaussian case the posterior distribution admits finite-dimensional sufficient statistics. There is a handful of special nonlinear situation when existence of finite-dimensional sufficient statistics for filtering density remains to be the case. However, in general this is not true, and one has to deal with infinite-dimensional posterior distributions (see section 2.1).

The customary way to address this problem is to compute the nonlinear filtering density by solving the Kushner or Zakai equations or in the case of discrete observations, the Fokker-Planck equation. As we show below, it is impossible to obtain corresponding solutions in real or almost real time using direct methods. Thus, special techniques are needed.

2 Summary of Results and Accomplishments

The main directions of the research and accomplishments of the project:

- Development of a spectral approach to optimal nonlinear filtering and related effective numerical approximations to the optimal nonlinear filter based on Wiener Chaos Expansion;
- A complete solution of “the last Wiener problem” – development of a Wiener type optimal nonlinear filter;
- Development of approximation methods for nonlinear filtering based on the projection filter and the assumed density filter;
- Nonlinear filtering with distributed observations;
- Clutter removal in nonlinear filtering for imaging data;
- Martingale problems for stochastic PDE’s;
- Spectral approach to parameter estimation in SPDE’s;
- Exponential forgetting of the initial condition for nonlinear (prediction) filters and their derivatives with respect to some parameter;
- Geometric ergodicity for the extended process whose components are the unobserved state process, the observation process, the prediction filter, and its derivative;
- Large time asymptotics (consistency, asymptotic normality) of parameter estimators in partially observed systems.

2.1 Numerical Approximations to the Optimal Nonlinear Filter Based on Wiener Chaos Expansions

The case where the state process, X_t , evolves in continuous time, while the observations, Y_t , are made in discrete moments $t = t_k$, $k = 0, 1, \dots$, is perhaps the most interesting case for applications. However, since in the case of continuous observation separation of parameters and observations is a more delicate problem, we present the results for this more difficult case. The approach is based on the technique of Wiener Chaos Expansions [62].

To be specific, let us consider the problem of estimation of a function of the state process, $f(X_t)$, at moment t based on observations $Y^t = \{Y_s, s \leq t\}$ (i.e., filtering) assuming that the signal and observation processes are described by the Itô stochastic differential equations

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t, \quad X_0 = x_0, \quad (1)$$

$$dY_t = h(X_t)dt + dV_t \quad (2)$$

where W_t and V_t are independent standard Brownian motions, $t \in R_+ = [0, \infty)$. The results may be extended to the case (important for some applications with “passive noises” like clutter, e.g.IRST systems, see section 2.3) where both the system noise and measurement noise are correlated processes and are correlated between each other. However, here for the sake of simplicity we consider simpler (still general enough) “uncorrelated” models (1), (2).

It is well known that the optimal mean square filtering estimate $\hat{f}_t = \mathbf{E}[f(X_t)|Y^t]$.

Define the process

$$\Lambda_t = \exp \left\{ \int_0^t h(\hat{X}_s) dY_s - \frac{1}{2} \int_0^t |h(\hat{X}_s)|^2 ds \right\}$$

where \hat{X}_s is a solution of the Itô equation

$$d\hat{X}_t = b(\hat{X}_t)dt + \sigma(\hat{X}_t)dW_t, \quad \hat{X}_0 = x_0.$$

Also, define the new probability measure $\tilde{\mathbf{P}}$ by $d\tilde{\mathbf{P}} = \Lambda_t^{-1}d\mathbf{P}$ and by $\tilde{\mathbf{E}}$ denote the operator of mathematical expectation with respect to this measure. Note that the measure $\tilde{\mathbf{P}}$ is a Wiener measure and the process Λ_t may be interpreted as a likelihood ratio process for the “signal+noise” and “noise only” models. Then usual Bayesian argument yields

$$\hat{f}_t = \tilde{\mathbf{E}}[f(X_t)\Lambda_t|Y^t] / \tilde{\mathbf{E}}[\Lambda_t|Y^t]$$

where $\phi_t[g] = \tilde{\mathbf{E}}[g(X_t)\Lambda_t|Y^t]$ is the so called unnormalized optimal filter.

Under very general assumptions, this filter is of the form

$$\tilde{\mathbf{E}}[f(X_t)\Lambda_t|Y^t] = \int_{\mathbf{R}^d} f(x) p_t(x) dx,$$

where the conditional probability density $p_t(x)$, usually referred to as the *unnormalized filtering density* (UFD), can be characterized as the unique solution of the following SPDE (Zakai equation):

$$dp_t(x) = \mathcal{L}^* p_t(x) dt + \sum_{k=1}^r h_k p_t(x) dY_t^k, \quad p_0(x) = \pi(x) \quad (3)$$

(see [15], [53], [64], [59]). Here we used the notation

$$\mathcal{L}^* = \sum_{i,j=1}^d a^{i,j} \frac{\partial^2}{\partial x_i \partial x_j} - \sum_{i=1}^d b^i \frac{\partial}{\partial x_i}$$

for the operator adjoint to the partial differential operator associated with (1) with covariance matrix $a = (a^{i,j}) = (1/2)\sigma\sigma^*$ (d and r are the dimensions of vectors X_t and Y_t , respectively), $\pi(x)$ is a “pdf” for the initial condition. We also assumed, without loss of generality, that the covariance matrix of measurement noise is a unit matrix.

Thus, the optimal nonlinear filtering problem reduces to computing nonlinear filtering density by solving the Zakai equation (3) or the Kushner equation (see [28]–[30]) or in the case of discrete observations, the Fokker-Planck equation.

Many applications, such as target tracking, require to solve these equations on-line, which is the most formidable obstacle on the way to real time implementation of optimal nonlinear filters. This problem is plagued by the curse of dimensionality. Specifically, if the spatial dimension $d \geq 3$, then the computational complexity of solving the second order parabolic PDE's like Fokker-Planck equation or stochastic PDE's as Kushner's or Zakai's equations becomes too high for direct on-line implementation.

On the first stage of our research supported by this grant, we proposed a spectral approach to nonlinear filtering, which allows us to circumvent the aforementioned computational difficulties by separating parameters and observations and shifting these time consuming operations off line (see [41]–[45], [49]). In the resulting algorithm, the on-line computations can be organized recursively in time and are relatively simple even when the dimension of the state process is large. Moreover, certain functionals of the state process can be estimated without computing the unnormalized filtering density. For the continuous time case, the approach is based on the Wiener Chaos Expansions and is explained below.

The main idea of the Wiener Chaos approach is to represent the unnormalized filtering density $p_t(x)$ in the form

$$p_t(x) = \sum_{\alpha} \phi^{\alpha}(r, t, x) \xi^{\alpha}(r, t) . \quad (4)$$

In this formula r is an arbitrary point of the interval $[0, t)$, the functions $\xi^{\alpha}(r, t)$ are normalized Wick polynomials (products of Hermit polynomials) of the Wiener integrals $\int_r^t m_k(s) dY_s$, where $\{m_k, k = 1, 2, \dots\}$ is a complete orthonormal system (CONS) in $L_2(0, t)$, $\phi^{\alpha}(r, t, x)$ are deterministic Fourier coefficients in the Cameron–Martin orthogonal decomposition of $p_t(x)$ and the summation in (4) is over all multi-indices α .

It is also proven in [40], [43], [49], [51], [52] (and this is the central part of the result) that $\phi_t^{\alpha}(x)$ satisfies a recurrent system of Kolmogorov-like equations. This system has an especially simple form if the observation process Y_t is one-dimensional. In this case, given a multi-index $\alpha = (\alpha_1, \alpha_2, \dots)$

$$\frac{\partial}{\partial s} \phi^{\alpha}(r, s, x) = \mathcal{L}^* \phi^{\alpha}(r, s, x) + \sum_k \alpha_k m_k(s) h^k(x) \phi^{\alpha(k)}(r, s, x) , \quad s > r \quad (5)$$

$$\phi^{\alpha}(r, r, x) = p_r(x) 1_{\{|\alpha|=0\}}$$

where \mathcal{L}^* is the same operator as in (3), and

$$\alpha(k) = (\alpha_1, \alpha_2, \dots, \alpha_{k-1}, \alpha_k - 1, \alpha_{k+1}, \dots) .$$

Below, system (5) is referred to as S -system.

The Wiener Chaos expansion for UFD (4) by itself does not provide yet the desired effect of separation. Indeed, on every time step, a solution of the S -system depends on the previous observation via its initial condition, and so could not be solved off line. To achieve complete separation, we consider the spatial Fourier coefficients of the UDF, $\psi_n(t) = \int_{\mathbf{R}^d} p_t(x) e_n(x) dx$, $n = 1, 2, \dots$

Let $0 = t_0 < t_1 < \dots < t_M = T$ be a uniform partition of the interval $[0, T]$ with step Δ (so that $t_i = i\Delta$, $i = 0, \dots, M$). Let $\{m_k^i\} = \{m_k^i(s)\}_{k \geq 1}$ be a CONS in $L_2([t_{i-1}, t_i])$. It was shown in [43] that the Fourier coefficients $\psi_n(t)$ satisfy the following recursive equation:

$$\psi_l(0) = \int_{\mathbf{R}^d} p(x) e_l(x) dx, \quad \psi_l(k) = \sum_n Q_{ln}(k) \psi_n(k-1), \quad k = 1, 2, \dots \quad (6)$$

where

$$\psi_l(k) = \psi_l(t_k), \quad Q_{ln}(k) = \sum_{\alpha} \Gamma_{l\alpha n} \xi^{\alpha}(t_{k-1}, t_k), \quad \Gamma_{l\alpha n} = \int_{\mathbf{R}^d} e_n(x) S_{\Delta}^{\alpha} e_l(x) dx$$

and where $S_{\Delta}^{\alpha} e_l$ is a solution of the S -system (5) subject to the initial condition $\phi^{\alpha}(0, 0, x) = e_l(x) 1_{\{|\alpha|=0\}}$.

Note that the kernel Γ is the only term in (6) that requires solving PDE's. This term is defined only by the parameters (the coefficients) of the state and observation processes (1), (2), and by the complete orthonormal system $\{e_l\}$, and so it can be precomputed off line. On the contrary, the Wick polynomials $\xi^{\alpha}(t_{i-1}, t_i)$ are defined only by the observation process $\{Y_t, t_{i-1} \leq t \leq t_i\}$ and the chosen orthonormal system $\{m_k^i\}_{k \geq 1}$. Hence (6) possesses the desired separation property.

The algorithm may be summarized as follows:

1. Before the observations become available

- a) compute $\psi_l(0) := (p, e_l)$ and $f_l = (f, e_l)$ where p is the initial density;
- b) compute $S_{\Delta}^{\alpha} e_l$, a solution of the S -system (5) subject to the initial condition $\phi^{\alpha}(0, 0, x) = e_l(x) 1_{\{|\alpha|=0\}}$;
- c) compute $\Gamma_{l\alpha n} = \int_{\mathbf{R}^d} e_n(x) S_{\Delta}^{\alpha} e_l(x) dx$.

2. When the observation become available (on the k -th step), $k = 1, 2, \dots$

- a) compute $\xi^{\alpha}(t_{k-1}, t_k)$;
- b) compute $\psi_l(k) = \sum_{\alpha} \sum_n \psi_n(k-1) \Gamma_{l\alpha n} \xi^{\alpha}(t_{k-1}, t_k)$;
- c) compute the UFD and the optimal filter,

$$p_{t_k} = \sum_l \psi_l(k) e_l(x) \quad (7)$$

$$\hat{f}_{t_k} = \sum_l \psi_l(k) e_l(x) / \sum_l \psi_l(k) (1, e_l). \quad (8)$$

We refer to this algorithm as the spectral separating scheme (S^3). Of course, to make S^3 computable one has to truncate all the involved infinite series. The convergence of the resulting approximation algorithm and a thorough error analysis has been done in [43], [51]. Here we only mention that even in the case of low order approximation based on Wick polynomials of order $|\alpha| = 2$ using only one element of the basis $\{m_k\}$ the mean square error of approximation for the UFD and the optimal filter is of the order of Δ^2 .

With the appropriate choice of bases the on-line computational complexity of S^3 is *linear*, i.e. the number of operations per step is $O(N_d)$ where N_d is the total number of spatial points (see [42]).

We would like to stress the following features of the algorithm:

- (1) The time consuming operations of solving the partial differential equation (5) and computing integrals are performed off line;
- (2) The overall amount of the off-line computations does not depend on the number of the on-line time steps;
- (3) Formula (8) can be used to compute an approximation to \hat{f}_t (e.g. conditional moments) without the time consuming computations of the filtering density $p_t(x)$ and the related integrals;
- (4) Only the Fourier coefficients ψ_l must be computed at every time step, while the approximate filter \hat{f}_t and/or the filtering density $p_t(x)$ can be computed as needed, e.g. at the final time moment.

Based on the principle of separation of observations and parameters, we developed a family of fast stochastic numerical algorithms for nonlinear filtering (see [42]–[44]). These numerical schemes were implemented and successfully tested in several problems of target tracking, including tracking of acutely maneuvering targets with angle-only measurements, fusion of kinematic (radar) tracking and imaging, etc.

2.2 Nonlinear Filtering with Distributed Observations

Filtering of a signal with distributed observation is one of the most important and at the same time most challenging problems of signal and image processing. A distinctive feature of this particular problem is that the observation is a sequence of random fields rather than a random process.

It was explained above that the main difficulty in practical implementation of nonlinear filters and predictors is their computational complexity. The main bulk of computations involved in nonlinear filtering comes from solving associated PDE's (Zakai or Kushner equations if the observation process is continuous in time and Fokker-Planck equation if

it is discrete). In the case of distributed observation the problem becomes even harder due to high dimensional measurements.

The spectral approach to nonlinear filtering developed by the PI and his collaborators in the setting with low dimensional observation can be extended to the case of distributed measurements. Moreover we argue that in addition to being temporally recursive, the resulting algorithm is also spatially recursive. More specifically, it can be shown that if at some point additional measurements become available, then they can be incorporated into the optimal filter without recomputing the latter from scratch. This property is very important since it allows to perform sequential multi-resolution filtering.

Example. An important practical motivation for the above setting is a problem of determining the position of a dim target moving in a plane or in 3D space, using a sequence of noisy images of the region of the space in which it evolves (see Figure 1).

If the signal-to-noise ratio (SNR) is large enough, the target could be localized on a single image using the well developed theory of matched filters (see Reed et al [58]). The problem of detection becomes much more difficult if localization of the target on a single image is impossible or at least hard and fraught with ambiguity (for example, the target may not be visible at all on any single frame, see Figure 2). In the latter case one has to align successive frames according to typical patterns of target dynamics. If the alignment is done properly the signals of the various images would add up and produce a “spike” with a sufficiently large SNR while the noises would cancel out. This approach to detection of a dim target is usually referred to as “tracking before detection” (TBD). Unfortunately the alignment of successive frames necessary for TBD is extremely difficult in the case of an acutely maneuvering noncooperative target.

To counter the aforementioned difficulty, an algorithm for frame alignment based on optimal nonlinear filtering and prediction was proposed in [26]. The results of implementation of the spatial-temporal nonlinear filtering algorithms with the use of Haar basis are shown in Figure 3. The target is completely localized after 30 frames of processing.

2.3 Clutter Removal In Nonlinear Filtering For Imaging Data

The majority of filtering methods used for target tracking and detection strongly rely on a “signal-plus-noise” (SPN) assumption for a mathematical model of initial data. However, the original sensor data typically does not fit such an assumption due to the presence of clutter components. Thus, an important issue is to eliminate the clutter and to reduce the original measurements to a SPN-model. To accomplish this goal, we consider nonparametric statistical methods.

Nonparametric regression algorithms can be regarded as methods of clutter estimation, or function smoothing, such that the residuals between the original data and its smoothed version, or estimate, would be reasonably approximated by SPN-models. Kernel methods provide a powerful tool for such analysis due to both computational transparency and asymptotic optimality in various settings of interest for observations in R^d with arbitrary

$d \geq 1$, see [17]. For example, if 2-D observations are of the form

$$Z_{ij} = f(x_i, y_j) + \xi_{ij}, \quad x_i = i/n_1, \quad j = j/n_2, \quad i = 1, \dots, n_1, j = 1, \dots, n_2,$$

where $f(x_i, y_j)$ is a value of an unknown function f at a point (x_i, y_j) , $\{\xi_{ij}\}$ are random variables with zero means, then a kernel estimator is given by

$$\hat{f}(x_i, y_j) = \frac{1}{N_1 N_2} \sum_{l_1, l_2} Z_{l_1 l_2} K\left(\frac{i - l_1}{N_1}, \frac{j - l_2}{N_2}\right),$$

where N_1, N_2 are window sizes in corresponding directions, $K(x, y)$ is a deterministic function, or kernel, $K : R^2 \rightarrow R^1$, such that $\int K(x, y) dx dy = 1$. It is readily seen that kernel estimators are weighted moving averages of observations.

The application of various kernels to real data sets is discussed in [37] along with the verification of "white noise" assumption for the model of residuals. The latter analysis relies on nonparametric rank methods [25]. The results of data smoothing and clutter removal are shown in Figure 4 a,b.

2.4 Martingale Problems for Stochastic PDE's

The results related to constructing the optimal nonlinear filters, reported above, would not have been possible without substantial development of the theory of stochastic PDE's. During the reporting period we made a substantial progress in this crucial area.

Specifically, we studied nonlinear stochastic PDE's with non-smooth (in some cases singular) coefficients. The examples include stochastic Navier-Stokes equation, Langevin (stochastic quantization) equation in $P(\varphi)_2$ Euclidean quantum field theory, SPDE's for the super-Brownian motion and some related superprocesses. We concentrated on existence, uniqueness, absolute continuity and singularity of distributions, and ergodicity problems for these equations.

Various classes of stochastic differential equations, where strong solutions do not exist or where their existence is very difficult to prove, can be handled using the martingale approach. For example, the martingale approach is very useful in situations with non-smooth coefficients typical of many SPDE's arising in physics and other sciences. These include the majority of equations obtained as limits of branching particle systems, equations of stochastic hydrodynamics, stochastic quantization equations in quantum field theory.

Our paper [48] deals with martingale problems in topological vector spaces and their applications for stochastic PDE's. In particular, we derive the necessary and sufficient criteria for absolute continuity of solutions to martingale problems. One interesting application of this result is the characterization of all the measures absolutely continuous with respect to super-Brownian motion. We also apply the obtained criteria to establish uniqueness and existence of certain classes of SPDE's including quasi-linear SPDE's

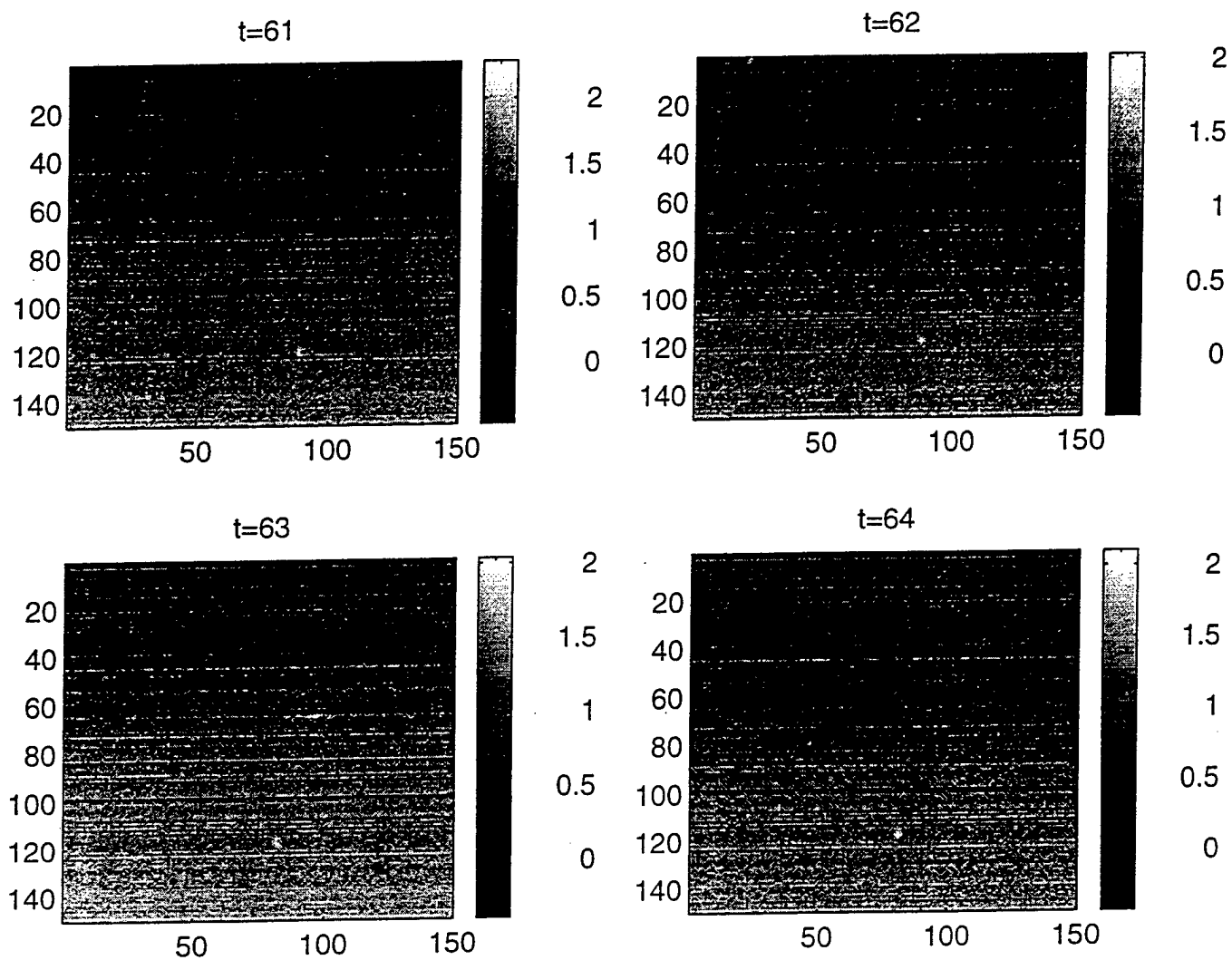


Fig. 1 Visible point target, Navy IRSS sensor- the LAPTEX field test

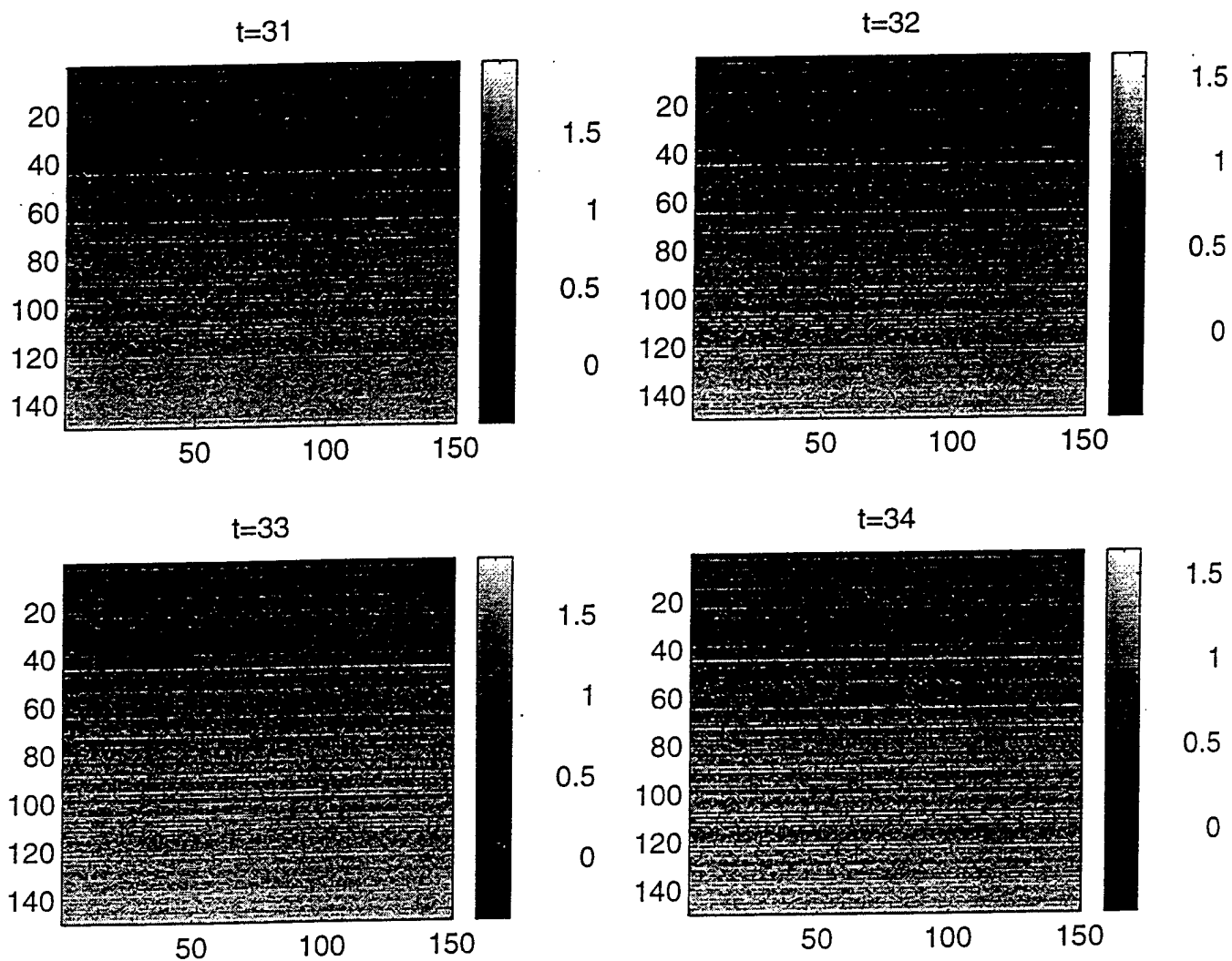


Fig. 2 Invisible point target, Navy IRSS sensor- the LAPTEX field test

Obs:140x140x66, Target:3x3, SNR -5.2dB
 Basis=Haar,128x128, CPU=0.66s/frame

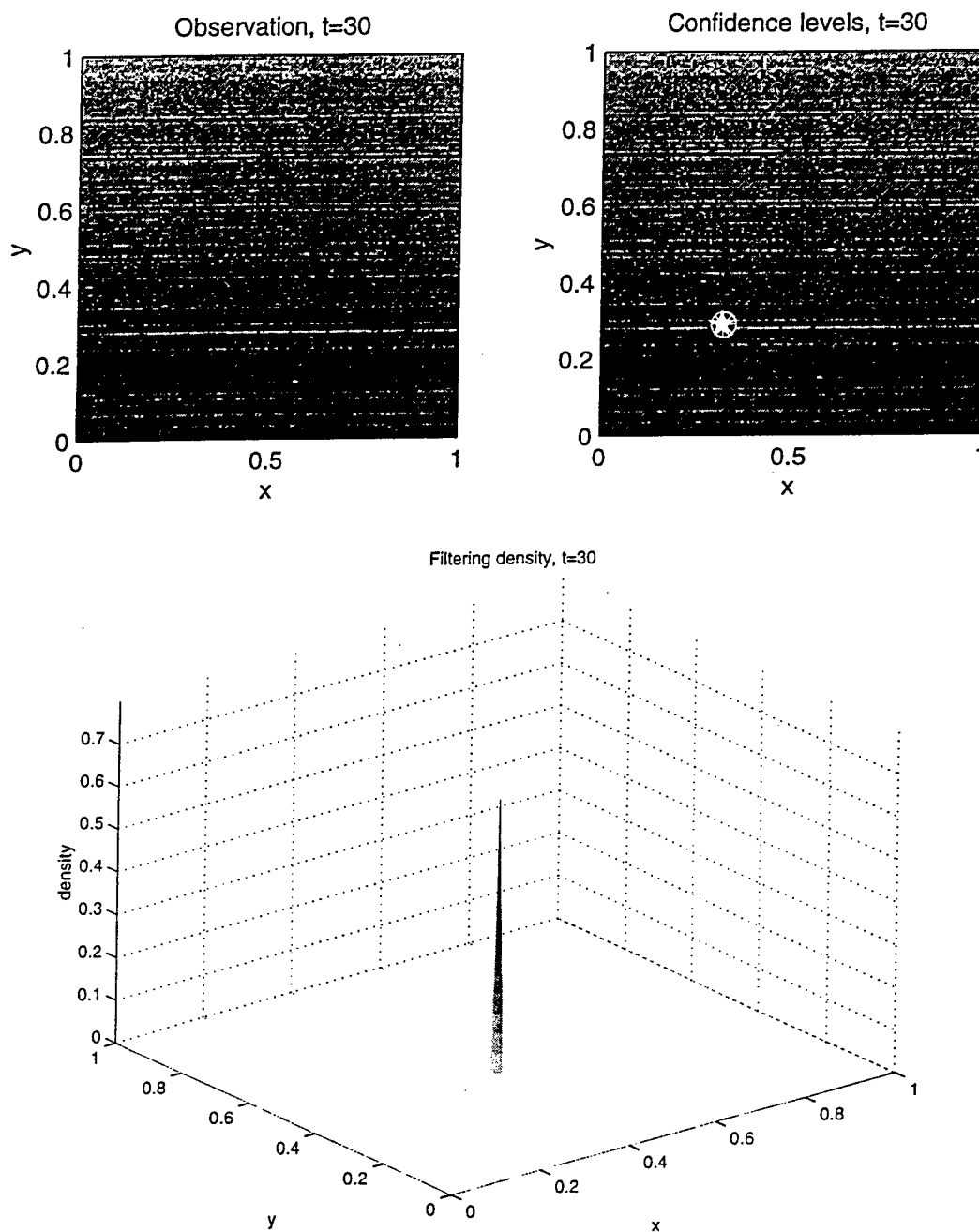


Figure 3: Results of off-line/on-line non-linear filtering, frame 30.

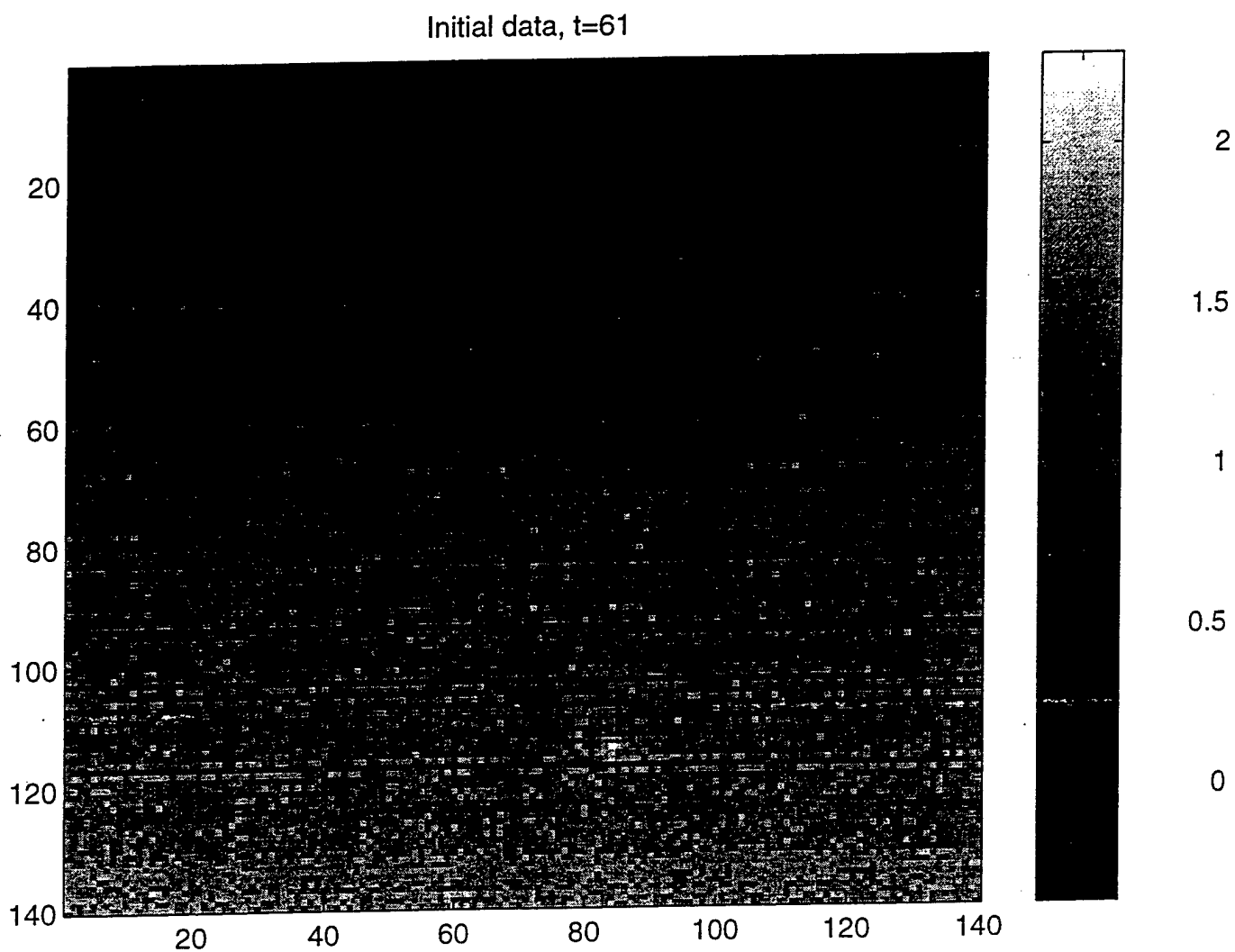


Fig. 4a Raw IR image, Navy IRSS sensor- the LAPTEX field test

Residuals after smoothing with Fuller kernels

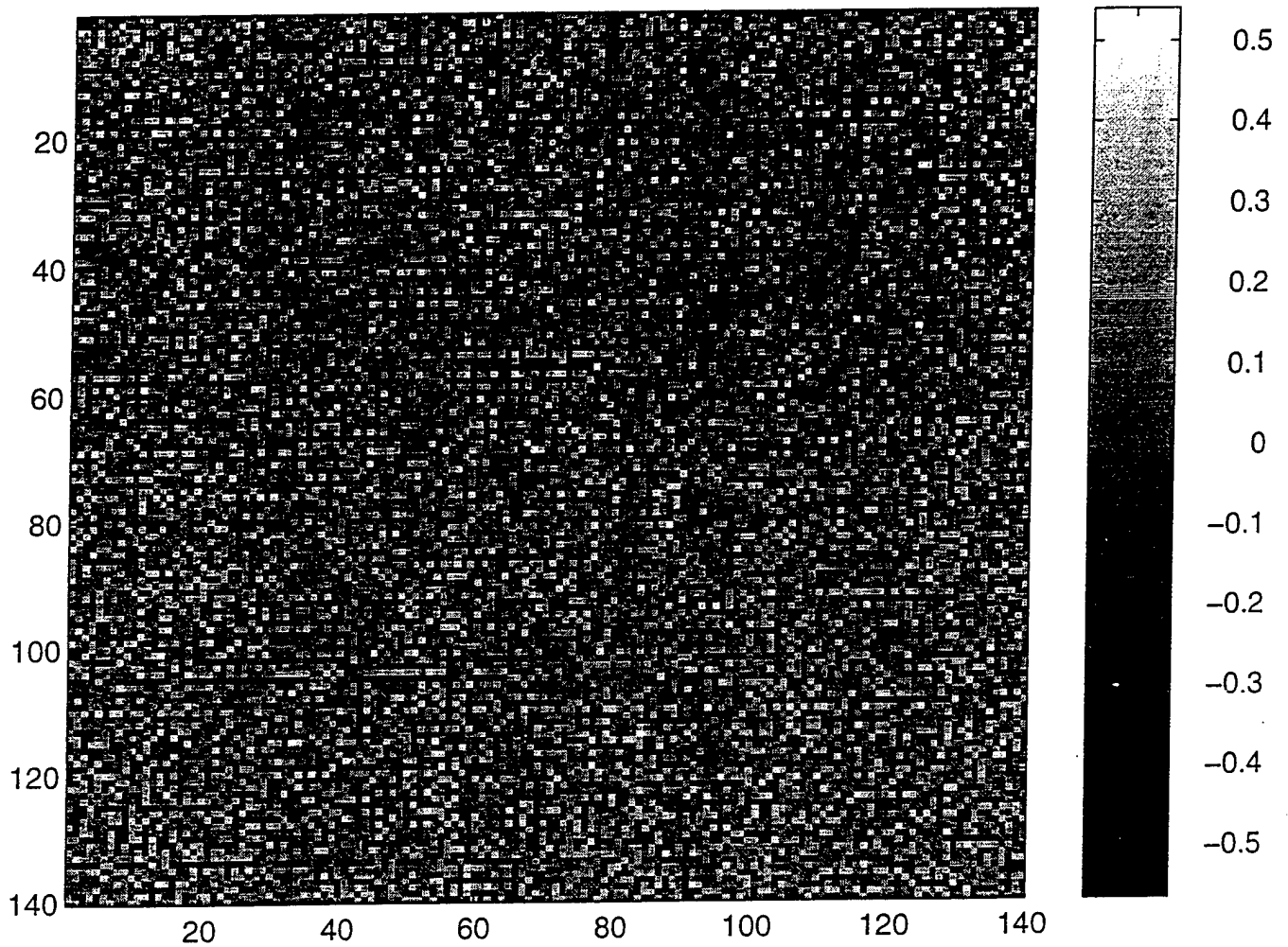


Fig. 4b Smoothed IR image, Navy IRSS sensor- the LAPTEX field test

with measurable (but not necessarily continuous) “drift”. In the same paper we extended Viot’s compactness method and applied it to solving nonlinear SPDE’s. Specifically, we investigated existence of weak solutions to stochastic Navier-Stokes type equations and some other quasi-linear SPDE’s with polynomially growing coefficients.

In [48] we also addressed the $P(\varphi)_2$ stochastic quantization equation. The equation of stochastic quantization was originally introduced and studied by physicists (see Parisi and Wu [56], Jona-Lasinsio and Mitter [24]) with the intent to bring dynamics in $P(\varphi)_2$ Euclidean quantum field theory. The idea was to demonstrate that this equation plays the role of Langevin’s equation, in that its solution is an ergodic Markov process whose unique invariant measure is the Euclidean $P(\varphi)_2$ measure. Due to the singular nature of the nonlinear term, the stochastic quantization equation is quite challenging. In [48] we proved the long standing conjecture that in the non-regular case the distributions of stationary solutions to this equation and the equation for the free field are singular.

2.5 Spectral Approach to Parameter Estimation in SPDE’s

Another important issue was the parameter estimation for stochastic evolution equation. In [18] and [46] we studied parameter estimation for randomly perturbed PDE’s. Specifically, we investigated asymptotic properties of estimators based on finite number of observable Fourier coefficients of the random field described by the PDE in question. Necessary and sufficient conditions were found for the consistency, asymptotic normality and efficiency of the estimates when the number of spatial modes increases.

These results were applied to some parametric models of passive scalars transport by turbulent flow. In particular, it was shown that the diffusivity exponent in the stochastic (advection-diffusion) equation for heat transport is always consistent, while the consistency of the feedback parameter estimate depends on the spatial dimension. Unlike previous works on the subject, no commutativity is assumed between the operators in the equation.

2.6 Projection Filter

Here we again consider a filtering problem where the state evolves according to a stochastic differential equation (SDE), and the objective is to estimate the state from nonlinear observations in additive Gaussian white noise (see (1), (2)). As we stated above, the filtering problem consists in the calculation of the whole conditional probability distribution of the state given past observations, which results in an infinite dimensional filter. Under some regularity assumptions, the conditional probability distribution is absolutely continuous with respect to the Lebesgue measure, and the conditional density p_t is the unique solution of the Kushner–Stratonovich equation, a stochastic PDE.

The projection filter is a finite dimensional nonlinear filter based on the differential geometric approach to statistics. In Brigo, Hanzon and LeGland [6, 9] the projection filter

was particularized to exponential families in the framework of SDE's on manifolds. The projection filter is defined by orthogonally projecting the right-hand side of the Kushner–Stratonovich equation for $\sqrt{p_t}$ onto the tangent space of a finite dimensional manifold of (square-root of) probability densities, according to the Fisher metric and its extension to infinite dimensional space of square roots of densities, known as the Hellinger distance. In practice, we use the manifold $EM^{1/2}(c)$ associated with an exponential family $EM(c)$ defined by the coefficients $c = \{c_1, \dots, c_m\}$. Although at first sight the resulting equation may look like a stochastic PDE, it is just a finite dimensional SDE for the parameter θ_t of the projection filter density $p(\cdot, \theta_t)$.

Another approximation method in nonlinear filtering is the assumed density filter (ADF) obtained by closing the equations for a few conditional moments, under the assumption that the exact conditional density is of a given form, see Kushner [31]. In the case of the exponential family $EM(c)$, one would

1. write the equation for the evolution of the conditional c -moment $\eta_t = \mathbf{E}[c(X_t) | \mathcal{Y}_t]$, and
2. evaluate all the conditional expectations appearing in the r.h.s. of the equation as if the conditional density was the density in the exponential family uniquely defined by the expectation parameter η_t .

The Itô-based ADF and the Stratonovich-based ADF are *different* filters in general, which is due to the *logical inconsistency* that is inherent to the ADF-concept: selecting a different set of equations to which it is applied, leads to different results.

Main results. We have shown in Brigo, Hanzon and LeGland [9] that it is possible to define simplifying exponential families $EM(c^*)$ and $EM(c^\bullet)$ such that the corresponding exponential projection filter has the following important properties:

- with the choice $EM(c^*)$, the diffusion coefficient in the stochastic differential equation for the PF parameter θ_t is constant, i.e. the equation has a very simple form, and it is possible to define an *a posteriori* estimate of the local error resulting from the projection filter approximation,
- with the choice $EM(c^\bullet)$, the correction step in the nonlinear filtering algorithm with discrete-time observations is handled exactly, without any error.

We have shown in Brigo, Hanzon and LeGland [8] that :

- for any exponential family $EM(c)$, the projection filter coincides with the Stratonovich-based assumed density filter,
- with the choice $EM(c^*)$, the Itô-based ADF coincides with the Stratonovich-based ADF.

2.7 Nonobservable Systems Observed in Small Noise

We have also studied the *small noise* asymptotics of the Bayesian estimator, based on continuous time nonlinear regression observation in additive Gaussian white noise

$$dX_t = m_t(\theta) dt + \varepsilon dW_t^\theta.$$

Using the Bayesian approach, we model the *a priori* information on the unknown parameter θ by a prior probability distribution, and the *posterior* probability distribution μ^ε is defined by the Bayes formula. Under the usual *identifiability* assumption that the true value α of the parameter is the only minimum point of the Kullback-Leibler information

$$K_\alpha(\theta) = \frac{1}{2} \int_0^T |m_t(\theta) - m_t(\alpha)|^2 dt,$$

the Bayesian estimator is consistent, i.e. $\mu^\varepsilon \Rightarrow \delta_\alpha$ in $\mathbf{P}_\alpha^\varepsilon$ -probability as $\varepsilon \downarrow 0$, see Kutoyants [32]. However, there are some practical situations where *nonidentifiability* occurs, i.e. where the set M_α of minimum Kullback-Leibler information does not reduce to $\{\alpha\}$. In this case, the Bayesian point estimator is not relevant, and we are rather interested in the asymptotic behavior of the *posterior* probability distribution μ^ε as $\varepsilon \downarrow 0$.

A prototype of this situation is the following problem. Consider a nonlinear filtering problem with noise-free dynamics, where the unobserved process evolves according to an ODE with unknown initial condition, and the observations are corrupted by a small additive Gaussian white noise. If the limiting deterministic system is nonobservable, then the corresponding statistical problem is nonidentifiable. Let us mention the TMA as a typical application where nonobservability occurs, see Lévine and Marino [38].

Main results. We have studied the case where the set M_α of points with minimum Kullback-Leibler information is a submanifold. It is easy to show that asymptotically as $\varepsilon \downarrow 0$ the probability distribution μ^ε is supported by M_α . We have shown that, using first order terms, such as the Fisher information matrix, it is possible to characterize the limit as $\varepsilon \downarrow 0$ of the probability distribution μ^ε , as a *random* probability distribution μ_α on M_α , absolutely continuous w.r.t. the canonical measure on M_α , and to provide an explicit expression for the density.

To study the rate of convergence, we have considered the *posterior* probability distribution ν^ε of the normalized deviation $[\theta - \pi(\theta)]/\varepsilon$, where π denotes the orthogonal projection on the set M_α . We have characterized the limit as $\varepsilon \downarrow 0$ of the probability distribution ν^ε as a mixture ν_α of *random* Gaussian probability distributions on the normal bundle space to M_α .

2.8 Exponential Forgetting and Geometric Ergodicity

Consider the situation where the state sequence $\{X_n\}$ is a time-homogeneous Markov chain with finite state space S , initial probability distribution $p_\bullet = (p_\bullet^i)$, and transition

probability matrix $Q_\bullet = (q_\bullet^{i,j})$. It is assumed that only observations $\{Y_n\}$ are available, which are mutually independent given the state sequence, with conditional densities $b_\bullet = (b_\bullet^i)$. In other words, we consider a typical hidden Markov model (HMM).

The prediction filter, which is the probability distribution of the state X_n given past observations Y_0, \dots, Y_{n-1} , solves an equation with values in the set $\mathcal{P}(S)$ of probability distributions over the finite set S . However in practice the initial probability distribution p_\bullet , the transition probability matrix Q_\bullet , and the vector b_\bullet of observation conditional densities – which define the model \mathbf{P}_\bullet – are generally unknown, and we consider instead the equation for the prediction filter $\{p_n\}$ associated with a wrong initial probability distribution p_0 , a wrong transition probability matrix Q , and a wrong vector b of observation conditional densities. Note that these misspecification issues are of a different nature:

- we expect that a wrong initial condition for the prediction filter is rapidly forgotten, so that we could use any initial condition with practically the same effect,
- on the other hand, we expect that two different transition probability matrices, and two different vectors of observation conditional densities will produce two significantly different observation sequences, so that we could estimate the unknown transition probability matrix and the unknown vector of observation conditional densities accurately, by accumulating observations.

Towards identification of HMM's, we consider also the gradient $\{w_n\}$ of the prediction filter $\{p_n\}$ w.r.t. some parameter, which solves a linear equation with values in the set $\Sigma = \{w : e^* w = 0\}$, which is the linear tangent space to (S) . Here $e = (1, \dots, 1)^*$ denotes the vector with all entries equal to 1.

Main results. Under the assumption that the transition probability matrix Q is *primitive*, we have obtained an explicit upper bound for the difference between the solutions of the misspecified prediction filter equation starting from two different initial conditions. We have also obtained an explicit bound for the Lipschitz constant of the solution map associated with the misspecified prediction filter equation.

These two non-logarithmic and non-asymptotic bounds go to zero at exponential rate as time goes to infinity, and as a consequence, under the additional assumption that the *true* transition probability matrix Q_\bullet is *primitive* as well, we have obtained an upper bound for the \mathbf{P}_\bullet -a.s. exponential rate of forgetting of the initial condition for the misspecified prediction filter equation.

We have obtained similar results about the forgetting of the initial condition for the misspecified linear tangent prediction filter.

Using the estimates on exponential forgetting, we have proved the geometric ergodicity of the extended Markov chain $\{Z_n = (X_n, Y_n, p_n, w_n)\}$, under some mild integrability assumption on the vectors b_\bullet and b of observation conditional densities.

As a consequence, we have also proved the uniqueness of an invariant measure, and the existence of a solution to the associated Poisson equation. From this result, the law of large numbers, and the central limit theorem can be proved for the extended Markov chain $\{Z_n = (X_n, Y_n, p_n, w_n)\}$.

2.9 Asymptotic Properties of the MLE and the CLSE

We consider the problem of HMM identification (i.e. partially observed finite-state Markov chain), based on noisy observations. We suppose that the law of the hidden Markov chain depends on some unknown finite-dimensional parameter $\theta \in \Theta$, and we are interested in the estimation of θ .

This is the continuation of the problem presented in the previous section. Along with the *true* but *unknown* model \mathbf{P}_\bullet , with initial probability distribution p_\bullet , transition probability matrix Q_\bullet , and vector b_\bullet of observation conditional densities, we consider a parametric model $\{\mathbf{P}^\theta, \theta \in \Theta\}$, with initial probability distribution $p_0 \neq p_\bullet$, transition probability matrix Q_θ , and vector b_θ of observation conditional densities.

We assume that there exists a *true* value $\alpha \in \Theta$ of the parameter, such that $Q_\bullet = Q_\alpha$ and $b_\bullet = b_\alpha$. Since the initial probability distributions p_\bullet and p_0 are possibly different, the *true* probability \mathbf{P}_\bullet does not belong in general to the family $\{\mathbf{P}^\theta, \theta \in \Theta\}$, i.e. the statistical model is *misspecified*. However, it follows from the results presented above that a wrong initial probability distribution is rapidly forgotten, so that from an asymptotic point of view, \mathbf{P}_\bullet and \mathbf{P}^α are practically equivalent. We make the assumption that for the *true* value $\alpha \in \Theta$, the transition probability matrix Q_α is *primitive*.

In this parametric model, it is easy to show that many functions of interest for the estimation of the unknown parameter θ , e.g. the log-likelihood function or the conditional least-squares functional, and the gradient of these functions w.r.t. the parameter θ , can be expressed as additive functionals of the extended Markov chain $\{Z_n^\theta = (X_n, Y_n, p_n^\theta, \partial p_n^\theta)\}$, where $\{p_n^\theta\}$ is the prediction filter corresponding to the value θ of the parameter, and $\{\partial p_n^\theta\}$ is the corresponding linear tangent prediction filter.

Main results. Using the explicit bound for the Lipschitz constant of the solution map associated with the prediction filter equation, we have proved that the log-likelihood function and the conditional least-squares functional are Lipschitz continuous w.r.t. the parameter, uniformly in time.

Using the existence of a solution to the Poisson equation associated with the extended Markov chain and the resulting law of large numbers and central limit theorem, we have been able

- to obtain an explicit expression for the \mathbf{P}_\bullet -a.s. limit of the log-likelihood function (suitably normalized), i.e. for the Kullback-Leibler information, and to prove that it is minimum in the true value α of the parameter;

- to prove the asymptotic normality of the score function (suitably normalized) and to obtain an explicit expression for the asymptotic covariance matrix, i.e. for the Fisher information matrix.

We have obtained similar results for the conditional least-squares functional.

These results allow us to prove

- the convergence of the maximum likelihood estimator and the conditional least-squares estimator to the set of maxima of the associated contrast function, using the Lipschitz continuity w.r.t. the parameter of the log-likelihood function and the conditional least-squares functional, respectively;
- the asymptotic normality of the maximum likelihood estimator and the conditional least-squares estimator.

We have obtained similar convergence results for recursive versions of these estimators, using the approach of Delyon [13] and Delyon and Iouditski [14].

3 Diffusion of Results

The results of our research on nonlinear filtering and SPDE's were presented by B. Rozovskii at several major conferences; these include: the 1996 *SIAM Annual meeting* (Kansas City), *Fourth World Congress of the Bernoulli Society* (Vienna, Austria), the 35th *IEEE Conference on Decision and Control* (Kobe, Japan), a mini-course on *Nonlinear Filtering* (Special Year in Stochastic Analysis, Mathematical Science Research Institute, Berkeley, 1997), invited talk at the *International Symposium on Stochastic Control and Nonlinear Filtering* (Los Angeles, 1997).

Other results have been presented by Marc Joannides at the HCM Workshop on *Statistical Inference for Stochastic Processes*, Sonderborg, Denmark, April 29–March 3, 1996, and at the Journées SMAI *Modélisation Aléatoire et Statistique (MAS)*, Toulouse, France, September 23–25, 1996; by François LeGland at the 2nd *Portuguese Conference on Automatic Control*, Porto, Portugal, September 11–13, 1996, at the Workshop on *Statistical Asymptotics for Continuous-Time Stochastic Processes*, LeMans, France, January 27–28, 1997, at the Workshop on *Stochastic Control and Nonlinear Filtering*, Raleigh, NC, October 11–12, 1996, at the Séminaire *Signal-Image*, IRISA, Rennes, France, November 28, 1996, at the International Symposium on *Mathematical Theory of Networks and Systems (MTNS)*, Saint Louis, MO, June 24–28, 1996, at the 36th *IEEE Conference on Decision and Control (CDC)*, San Diego, December 10–12, 1997 and at the symposium on *Stochastic Control and Nonlinear Filtering*, Los Angeles, December 13–15, 1997; by Laurent Mevel at the workshop on *Hidden Markov Models*, Evry, France, April 24, 1997, and at the 4th *European Control Conference (ECC)*, Brussels, July 1–4, 1997.

4 List of Publications and Technical Reports

1. D. Brigo, *Filtering by Projection on the Manifold of Exponential Densities*. PhD Thesis, Vrije Universiteit, Amsterdam, November 1996.
2. D. Brigo, B. Hanzon, and F. Le Gland, "A differential geometric approach to nonlinear filtering: the projection filter," In *Proceedings of the 34th Conference on Decision and Control*, New Orleans 1995, pp. 4006-4011. IEEE-CSS, December 1995.
3. D. Brigo, B. Hanzon, and F. Le Gland, "The exponential projection filter and the selection of the exponential family," In *Proceedings of the 2nd Portuguese Conference on Automatic Control*, Porto 1996, pp. 251-256. APCA, September 1996.
4. D. Brigo, B. Hanzon, and F. Le Gland, "On the relationship between assumed density filters and projection filters," Discussion Paper TI 7-96-18, Tinbergen Institute, February 1996.
5. D. Brigo, B. Hanzon, and F. Le Gland, "A differential geometric approach to nonlinear filtering: the projection filter," *IEEE Transactions on Automatic Control*, vol. AC-43, no. 2, pp. 247-252, February 1998.
6. D. Brigo, B. Hanzon, and F. Le Gland, "A differential geometric approach to nonlinear filtering: the projection filter," *IEEE Transactions on Automatic Control*, vol. AC-43, no. 2, pp. 247-252, February 1998.
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8. Z. Cai, F. Le Gland, and H. Zhang, "An adaptive local grid refinement method for nonlinear filtering," Publication Interne 954, IRISA, October 1995.
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10. M. Joannides and F. Le Gland, "Nonlinear filtering with perfect discrete time observations," In *Proceedings of the 34th Conference on Decision and Control*, New Orleans 1995, pp. 4012-4017. IEEE-CSS, December 1995.
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12. M. Joannides, *Navigation Intégrée d'un Engin Sous-Marin Remorqué. Filtrage Non-Linéaire des Systèmes avec Contraintes et/ou Mesures Parfaites*. Thèse de Doctorat, Université de Provence, Marseille, March 1997.
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14. M. Joannides and F. Le Gland, "Small noise asymptotics of nonlinear filters with nonobservable limiting deterministic system," In *Proceedings of the 36th Conference on Decision and Control, San Diego, 1997*, pp. 1663-1668. IEEE-CSS, December 1997.
15. M. Joannides and F. Le Gland, "Small noise asymptotics of nonlinear filters with nonobservable limiting deterministic system," submitted to *Statistical Inference for Stochastic Processes (SISP)*.
16. F. Le Gland and L. Mevel, "Asymptotic behaviour of the MLE in hidden Markov models," In *Proceedings of the 4th European Control Conference, Bruxelles 1997*, July 1997. Paper FRA-F6.
17. F. Le Gland and L. Mevel, "Exponential forgetting and geometric ergodicity in HMM's," In *Proceedings of the 36th Conference on Decision and Control, San Diego 1997*, pp. 537-542. IEEE-CSS, December 1997.
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19. F. Le Gland and L. Mevel, "Exponential forgetting and geometric ergodicity in hidden Markov models", submitted to *Mathematics of Control, Signals and Systems (MCSS)*.
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31. M. Hübner, S.V. Lototsky, and B.L. Rozovskii, "Asymptotic properties of an approximate maximum likelihood estimator for stochastic PDE's," In: *Statistics and Control of Stochastic Processes, The Liptser Festschrift* (Ed., Yu. Kabanov, B. Rozovskii, A. Shiryaev), World Scientific Publ. Co., 1997.
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5 Scientific Personnel Supported by This Project

- D. Brigo (INRIA – Ph.D. in Applied Mathematics from Vrije Universiteit, Amsterdam, 1996)
- F. Campillo (INRIA)
- M. Joannides (INRIA – Ph.D. from Universite de Provence, Marseilles, 1997)
- S. Kligys (USC)
- F. Le Gland (INRIA)
- R. Liptser (Tel Aviv University, consultant)
- J. Li (USC)
- S. Lototsky (USC – Ph.D. in Applied Mathematics from USC, 1996)
- L. Mevel (INRIA – Ph.D. from Universite de Rennes 1, Rennes, 1997)
- R. Mikulevicius (USC)
- E. Remi (INRIA)
- B. Rozovskii (USC)
- A. Tartakovsky (USC)
- H. Zhang (INRIA)

6 Report of Inventions

“Method of Optimal Stochastic Filtering for Tracking Objects with Possibly Numerical Dynamics”

7 Technology Transfer

The developed nonlinear filtering technology (particularly, track-before-detect algorithm based on this technology) is in the process of transfer to the small business company as well as to the TRW Data Technologies Division (Systems Integration Group) under the 1998 SBIR-STTR program. It is expected that the developed ideas will be used in both radar and electro-optical warning systems in BMDO programs.

Aegir Systems (Oxnard, CA) is applying our numerical S^3 algorithm for the development of a tracking filter. Testing of Aegir's tracker indicates that the advantage of the proposed approach is especially significant for angle-only tracking of resolved or sub-resolved targets (IR, EO search and track systems) when signal-to-noise ratio is extremely low (-3dB to -6dB).

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